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A note on the Ramsey number of stars — Complete graphs

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ABSTRACT

Boza et al. [L. Boza, M. Cera, P. García-Vázquez, M.P. Revuelta, On the Ramsey numbers for stars versus complete graphs, European J. Combin. 31 (2010) 1680–1688] gave the exact value of the multicolor Ramsey number $R(K_{1,q_1}, \ldots, K_{1,q_n}, K_{p_1}, \ldots, K_{p_m})$ in terms of $R(K_{p_1}, \ldots, K_{p_m})$. In this note, we give a short proof of this result.

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1. Introduction

For given graphs $G_1, G_2, \ldots, G_t$, the *multicolor Ramsey number* $R(G_1, G_2, \ldots, G_t)$ is the smallest positive integer n such that, if the edges of a complete graph K_n are partitioned into t disjoint color classes giving t graphs $H_1, H_2, \ldots, H_t$, then at least one H_i has a subgraph isomorphic to G_i. The multicolor Ramsey number of stars was completely determined by Burr and Roberts [2]. Moreover, Jacobson [4] determined the value of the multicolor Ramsey number of stars versus a complete graph as follows.

Theorem 1.1 ([4]). *Let $p, q_1, \ldots, q_n$ be positive integers and $r = R(K_{1,q_1}, \ldots, K_{1,q_n})$. Then $R(K_{1,q_1}, \ldots, K_{1,q_n}, K_p) = (r-1)(p-1)+1$.*

Boza et al. extended Theorem 1.1 as follows.

Theorem 1.2 ([1]). *Let $p_1, \ldots, p_m, q_1, \ldots, q_n$ be positive integers, $r = R(K_{1,q_1}, \ldots, K_{1,q_n})$ and $r' = R(K_{p_1}, \ldots, K_{p_m})$. Then $R(K_{1,q_1}, \ldots, K_{1,q_n}, K_{p_1}, \ldots, K_{p_m}) = (r-1)(r'-1)+1$.*

In this note, we give a short proof of Theorem 1.2. Moreover, the multicolor Ramsey number $R(C_n, K_{p_1}, \ldots, K_{p_m})$ is determined in terms of $r = R(K_{p_1}, \ldots, K_{p_m})$ when $n \geq 4r+2$.

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2. The proof

Theorem 1.2 is an easy consequence of Theorem 1.1 and the following result.

Theorem 2.1. *Let $G_1, G_2, \ldots, G_n$ be connected graphs, $r = R(G_1, G_2, \ldots, G_n)$ and also $r' = R(K_{p_1}, \ldots, K_{p_m})$. If $l \geq 2$ and $R(G_1, \ldots, G_n, K_l) = (r-1)(l-1)+1$, then*

$$R(G_1, \ldots, G_n, K_{p_1}, \ldots, K_{p_m}) = (r-1)(r'-1)+1.$$

Proof. Let $R = R(G_1, \ldots, G_n, K_{p_1}, \ldots, K_{p_m})$. For the lower bound, begin with an m-coloring of the edges of $K_{r'-1}$, say colors $\beta_1, \ldots, \beta_m$, that has no copy of K_{p_i} in color β_i for any i, $1 \leq i \leq m$. Replace each vertex of $K_{r'-1}$ by a complete graph of order $r-1$ whose edges are colored by colors $\alpha_1, \ldots, \alpha_n$ such that the i-th color class contains no copy of G_i, $1 \leq i \leq n$. Each edge in the original graph $K_{r'-1}$ expands into a copy of $K_{r-1,r-1}$. Give all edges in this subgraph the same color that its original edge had. This yields an $(m+n)$-edge coloring of $K_{(r-1)(r'-1)}$ with colors $\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_m$ which contains no copy of G_i in color α_i, $1 \leq i \leq n$, and no K_{p_j} in color β_j, $1 \leq j \leq m$. This means that $R \geq (r-1)(r'-1)+1$.

Now, let c be an arbitrary $(m+n)$-edge coloring of the complete graph $G = K_{(r-1)(r'-1)+1}$ by colors $\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_m$. Recolor the edges of colors $\beta_1, \ldots, \beta_m$ by a new color α and retain the color of the remaining edges. This yields an edge coloring of G by colors $\alpha_1, \ldots, \alpha_n, \alpha$. Since $R(G_1, G_2, \ldots, G_n, K_{r'}) = (r'-1)(r-1)+1$, G contains a copy of G_i of color α_i for some $1 \leq i \leq n$, or a copy of $K_{r'}$ of color α. If the first case occurs, we are done, and otherwise we have a monochromatic copy of $K_{r'}$ in color α. Return to c, restricted to this set of r' vertices. The definition of the Ramsey number says that c has a monochromatic copy of K_{p_j} in color β_j on these vertices, for some $1 \leq j \leq m$. This observation completes the proof. $\square$

In [3], Erdős et al. conjectured that $R(C_n, K_l) = (n-1)(l-1)+1$ for every $n \geq l \geq 3$, except for $n = l = 3$. This conjecture has been proved by Nikiforov for $n \geq 4l+2$ (see [5]). Using Theorem 2.1, we can generalize this result as follows.

Corollary 2.2. *$R(C_n, K_{p_1}, \ldots, K_{p_m}) = (n-1)(r-1)+1$, where $r = R(K_{p_1}, \ldots, K_{p_m})$ and $n \geq 4r+2$.*

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